Int. J. Heat Mass Transfer. Vol. 19, pp. 765–770. Pergamon Press 1976. Printed in Great Britain

THE EVALUATION OF A ONE-DIMENSIONAL TEMPERATURE PROFILE IN A TURBULENT FLUID FLOW FROM THE AMPLITUDE DISTRIBUTION OF MEASURED TEMPERATURE FLUCTUATIONS AT A FIXED DETECTOR POSITION

E. OHLMER and D. SCHWALM
C.C.R. Euratom, 21020 Ispra (Va), Italy

(*Received* 28 *May* 1975)

Abstract—In a turbulent homogeneous fluid flow temperature fluctuations are generated by mixing of the fluid with a lateral temperature profile. In case of a flow with a one-dimensional temperature profile in a direction perpendicular to the flow velocity, the amplitude distribution of the temperature noise is Gaussian if this profile is linear, independently of the numerical value of the gradient. Deviations from the linear profile cause deviations from the normal distribution. Due to this fact it will be possible to calculate the underlying one-dimensional profile from the probability density function of the noise measured at a fixed position in the fluid flow.

Experiments in a homogeneous turbulent water flow ($Re = 27\,000$) confirmed the described method.

The evaluation of a temperature profile by only one fixed noise detector may be of practical interest for malfunction detection in fuel elements of nuclear power reactors of the LMFBR-type.

NOMENCLATURE

- a, gradient of a linear temperature profile;
- D, lateral detection range;
- P, cumulative probability distribution function;
- p, probability density function;
- Re, Reynolds number;
- T, temperature;
- t, time;
- X, displacement;
- $\mathbf{x}$, three-dimensional vector;
- x, y, lateral coordinates;
- z, coordinate in direction of the flow.

Greek symbols

- δ, scaling factor;
- σ, root mean square value (RMS);
- Θ, temperature range.

INTRODUCTION

THE TEMPERATURE profile in a fluid flow will usually be measured by thermocouples placed at different positions vertical to the flow direction. As a result the temperature profile can be plotted at discrete coordinate values. This standard technique will be applied in case of laminar and turbulent flow.

In turbulent homogeneous flow however, it is not necessary to measure the temperature at different lateral positions in order to get the temperature profile in a certain range. It is the aim of this paper to show that it is sufficient to measure the temperature fluctuations and the average temperature at only one fixed position in the flow.

In Part I we investigate analytically the influence of the shape of the temperature profile on the amplitude distribution of the temperature fluctuations. It will be possible to derive a relation between the fluctuation intensity and the gradient of a linear temperature profile. In addition a method will be described for the calculation of the profile from the cumulative distribution function of the temperature noise measured at a fixed position in the fluid flow.

In Part II this method will be applied to temperature noise measured in a turbulent water flow.

I. THEORETICAL BACKGROUND

I.1. *The influence of the temperature profile on the amplitude distribution*

It is well known from the literature, e.g. [1, 2] that temperature noise measured in a water flow may be regarded as almost Gaussian. The measured deviations from this normal distribution were interpreted as caused by a large gradient in the temperature profile [2, 3]. It is intended to investigate the relation between the amplitude distribution of the temperature noise and the underlying temperature profile.

We assume a homogeneous turbulent fluid flow in the z-direction with a lateral temperature profile $T(x)$ which is independent of the coordinate y. Now we consider a "marked" fluid particle (e.g. a temperature eddy) at time t_0 at position x which will be dispersed by the turbulent motion of the fluid in such a way that it reaches position x' at time t. For convenience we ignore the effect of molecular diffusion. Following Batchelor and Townsend [4] we define a probability density function $p\,(\mathbf{x}, t_0, \mathbf{x}', t)$ of the displacement $\mathbf{x}' - \mathbf{x}$ for this "marked" fluid particle.

In the case of homogeneous turbulence in the lateral direction x the probability density function p is a function of the displacement $X = x - x'$ of the particle after time $t - t_0$, where x and x' are the scalar components of the vectors $\mathbf{x}$ and $\mathbf{x}'$ in the x-direction. If we now assume that vector $\mathbf{x}'$ lies on the z-axis, the diplacement X will be equal to the coordinate x. According to [4] the probability density function of the displacement $x(t)$ is asymptotically normal, i.e.

$$p(x) = \frac{1}{\sqrt{(2\pi)} \cdot \sigma_x} \exp\left[-\frac{x^2}{2\sigma_x^2} \right] \tag{1}$$

with $\sigma_x = \sqrt{(\overline{x^2})}$ (RMS-value of the displacement) when $t - t_0$ is large.

In case that the fluid flow has a stationary temperature profile $T(x)$, the fluid particles (temperature eddies) at a certain distance x from the z-axis are "marked" in the mean by the temperature $T(x)$. The probability density function $p(T)$ which is given by the probability density function $p(x)$ and the temperature profile $T(x)$ by insertion of the inverse function $x(T)$ into relation (1). As σ_x depends on the turbulent character of the fluid flow, it remains essentially unchanged by the temperature profile.

In the case of a linear temperature profile $T(x) = ax$ it follows that

$$p(T) = \frac{1}{\sqrt{(2\pi)}} \cdot \frac{1}{a\sigma_x} \cdot \exp\left[-\frac{T^2}{2(\sigma_x a)^2} \right] \tag{2}$$

satisfying condition

$$\int_{-\infty}^{\infty} p(T)\,\mathrm{d}T = 1 \tag{3}$$

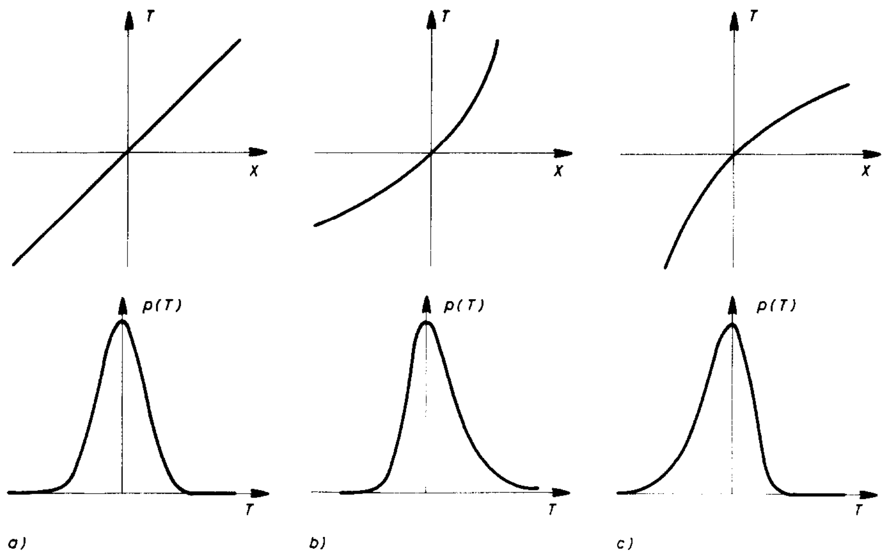

FIG. 1. The influence of the temperature profile $T(x)$ on the probability density function $p(T)$: (a) linear profile; (b) convex profile; (c) concave profile.

turbulent mixing of these temperature eddies with different temperatures causes fluctuations which might be measured by a detector positioned at a point on the z-axis. Or, in other words, the temperature fluctuations at this fixed point is a time series of alternating "hot" and "cold" lumps of fluid of constant temperature [5]. Hence it may be concluded that temperature noise is generated by the turbulent mixing of a fluid with a gradient in the temperature profile [6, 7].

As we consider "marked" particles starting from points on the x-axis we have to place a detector at that distance from the x-axis where in the mean the temperature eddies did not lose their identity. This distance will be given by the so called "coherence length". It can be evaluated by cross correlation measurements which will be described in a following paper. Some preliminary results indicate that this length is of such an order that the lateral temperature profile remains practically unchanged within this range down-stream the fluid flow.

The temperature fluctuations measured by the detector placed on the z-axis will consequently have a probability density function $p(T)$. Hence $p(T)\mathrm{d}T$ represents the probability that the measured random temperature fluctuation $T(t)$ falls in the interval $T < T(t) < T + \mathrm{d}T$.

As a consequence it follows that the RMS-value of the temperature fluctuations $\sigma_T = \sqrt{(\overline{T^2})}$ is given by

$$\sigma_T = \frac{\mathrm{d}T}{\mathrm{d}x} \cdot \sigma_x, \tag{4}$$

it means that the intensity of the temperature noise is proportional to the gradient of the linear temperature profile. This basically corresponds with published results, see e.g. [3].

In addition to the case of a linear profile we also calculated the resulting functions $p(T)$ for a convex and a concave profile as illustrated in Fig. 1. As a result it may be stated that the $p(T)$-function is Gaussian only in the case of a linear temperature profile. A deviation from the normal distribution is caused by a space-dependent gradient of the temperature profile.

In the literature [2, 3] it was stated that a deviation from the normal distribution is caused by a large

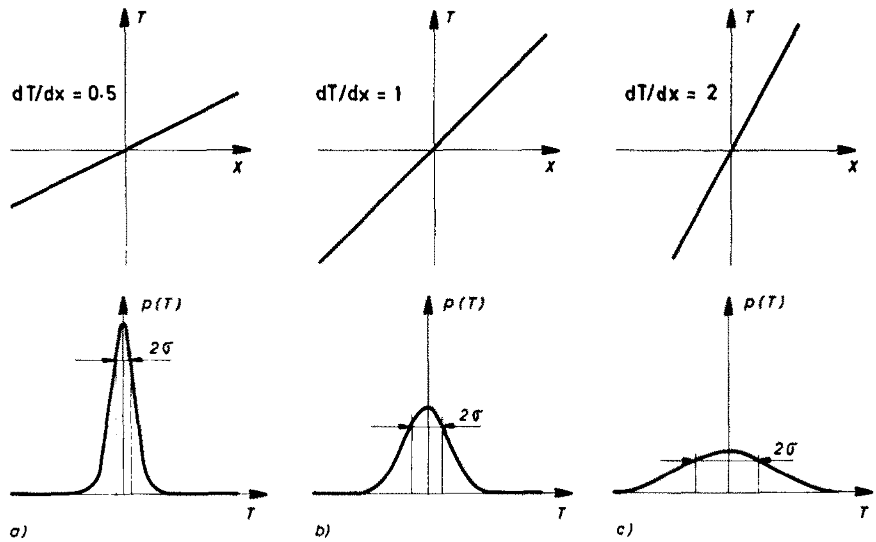

FIG. 2. The dependence of the fluctuation intensity σ_T on the gradient of a linear temperature profile: (a) $dT/dx = 0.5$; (b) $dT/dx = 1$; (c) $dT/dx = 2$.

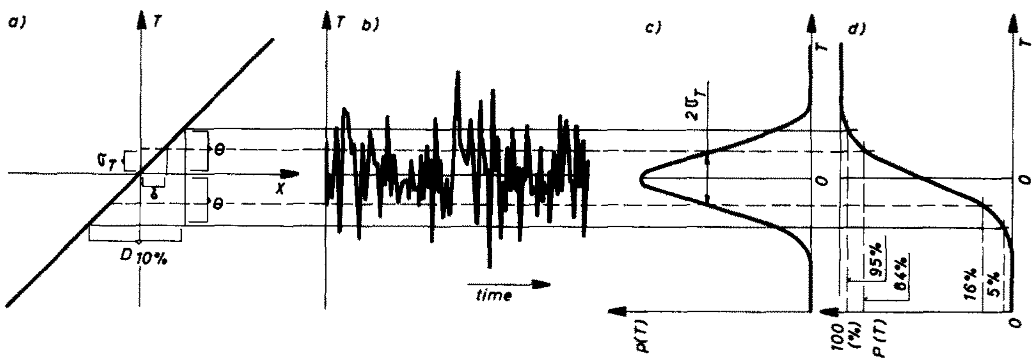

FIG. 3. Normal distributed temperature noise: (a) underlying lateral temperature profile $T(x)$; (b) temperature noise signal $T(t)$; (c) probability density function $p(T)$; (d) cumulative probability distribution function $P(T)$.

temperature gradient. Our results do not confirm this statement. On the contrary it follows from relation (2) that $p(T)$ is Gaussian independently of the numerical value of the constant gradient dT/dx. Figure 2 shows the influence of the constant gradient on $p(T)$ and σ_T, respectively.

I.2. *The lateral detection range of a detector*

According to relation (1) the probability for a temperature eddy to reach the z-axis starting its turbulent motion at distance x decreases with increasing x. At a certain distance $x = D/2$ the probability, that an eddy reaches the detector on the z-axis, is practically zero. This means, that the detector can "see" temperature eddies starting only from points inside the zone $-D/2 \leqslant x \leqslant D/2$. Hence this zone may be regarded as the detection range D.

In order to give a more precise meaning of this detection range, it is more convenient to look for a certain percentage of temperature eddies which could be detected. Regarding the cumulative probability distribution function $P(T)$

$$P(T) = \int_{-\infty}^{T} p(T)\,dT \tag{5}$$

and the linear temperature profile, as illustrated in Fig. 3, we consider, e.g. the case that 10% of the temperature noise amplitudes may have values greater than Θ and smaller than −Θ. From curves (a) and (c) of Fig. 3 it follows that 10% of the detected temperature eddies are coming from points outside the range

$$-\tfrac{1}{2}D_{10\%} \leqslant x \leqslant +\tfrac{1}{2}D_{10\%}.$$

The extent of the total detection range depends on the statistical accuracy of the analysis, i.e. on the total length of the processed signal.

In the following section we further need a range 2δ where Θ is equal to the RMS-value σ_T. Hence δ follows from curves (a) and (c) for $P(\sigma_T) = 0.8413$. According to this definition and to relation (4) it follows that δ is equal to σ_x.

I.3. *The determination of the temperature profile from a measured cumulative distribution function* P(T)

From the measured noise $p(T)$ will be evaluated at first and consecutively $P(T)$ will be calculated. This function can be transformed into a function $T(\sigma_T)$ by taking the temperature values from function $P(T)$ at percentage values of P corresponding to discrete values of σ_T.

In the case of Gaussian noise $T(\sigma_T)$ will be a linear function. In case of non-Gaussian noise $T(\sigma_T)$ will have a shape similar to the temperature profile $T(x)$. According to relation (4) there is an equivalence between σ_T (°C) and δ (cm).

As outlined above, this particular detection range δ is a characteristic parameter of the turbulent flow. It can be evaluated experimentally from a known linear lateral temperature profile in the fluid flow and the measurement of σ_T. Now, replacing σ_T by this scale factor δ the function $T(\sigma_T)$ will be transformed into the real profile $T(x)$.

Up to now we assumed that the detector measures only the temperature fluctuations, i.e. that the mean temperature at the detector position was set equal to zero. As a consequence the calculated temperature profile is zero at $x = 0$. However in practice this relative profile must be shifted by the mean temperature value at the detector position. Hence it is necessary to measure also at this point the mean temperature.

An experimental verification of this method will be described in Part II.

II. EXPERIMENTAL VERIFICATION BY TEMPERATURE NOISE MEASUREMENTS

II.1. *The test section*

The temperature fluctuation signals we used for the verification were obtained from measurements at a test section constructed for other investigations in course [8]. In a rectangular channel with a 20.5 × 3 cm cross section a temperature profile was generated in a water flow by means of a row of heater pins (diameter 2.0 cm) in the inlet region. The resulting profile has a minimum value of about 12°C (inlet temperature) and a maximal value of about 14°C at a flow velocity of 45 cm/s ($Re = 27\,000$).

Due to the turbulent mixing of the fluid with this profile temperature fluctuations were generated which were measured by 4 detectors in the channel at a distance of 65 cm downstream from the end of the heater pins. These detectors were mounted in the channel with a lateral distance of 2.5 cm from each other.

II.2. *Instrumentation and data processing*

Two kinds of temperature detectors were used. The mean temperature of the profile was measured by normal thermocouples placed between the noise detectors. The temperature fluctuations however were measured by special detectors which are sensitive only to temperature fluctuations and automatically remove the mean temperature from the signal [9]. These signals were amplified by a low noise amplifier by a factor of 10^4.

This instrumentation was chosen for the originally planned measurements and therefore not optimized for the measurements analysed in this paper.

As outlined in Part I we had to measure the temprature fluctuations and the mean temperature at a fixed position in the fluid flow. This could be done by a normal thermocouple. However, as the fluctuating component of the temperature signal is much smaller than the mean value, it is better to amplify these two components separately. This will be possible with the special detectors developed in the JRC Ispra. Figure 4 shows such a detector and in addition the detectors used in this experiment.

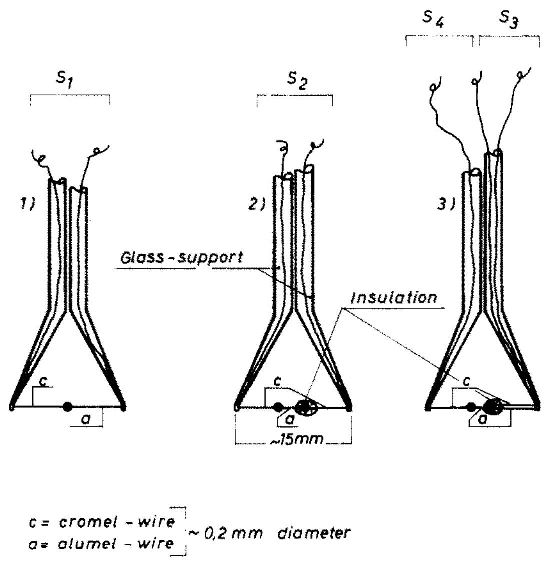

FIG. 4. Schematic view of temperature detectors: (1) usual thermocouple; signal $S_1 = \bar{T} + T(t)$; (2) noise detector; signal $S_2 = T(t)$; (3) combined detector; signals $S_3 = \bar{T}$ and $S_4 = T(t)$. Types (1) and (2) were used in this experiment.

The signals of the fluctuation detectors were recorded on a magnetic tape and processed by a Time Data statistical analyzer TD 1923/30.

In order to eliminate the 50 Hz hum the signals were filtered with a low-pass filter at 35 Hz though they contained useful information up to 80 Hz.

In order to reduce the statistical error 80 independent signals of 5.12 s length were processed. This means that the averages were evaluated from a signal of 7 min length. In addition to the probability density function $p(T)$ all the other calculations were performed with the analyzer by programmes written in TSL.*

Thus the temperature profiles are calculated in a few seconds.

II.3. *The measured temperature noise*

The probability density functions $p(T)$ of the four noise detectors are plotted in Fig. 5 in the upper line. The graphs in the lower line of this figure show the four temperature profiles $T(x)$ calculated by this method. The detection range δ has been evaluated to

*TSL is the Time Series Language developed by the Time Data Company for time series analysis.

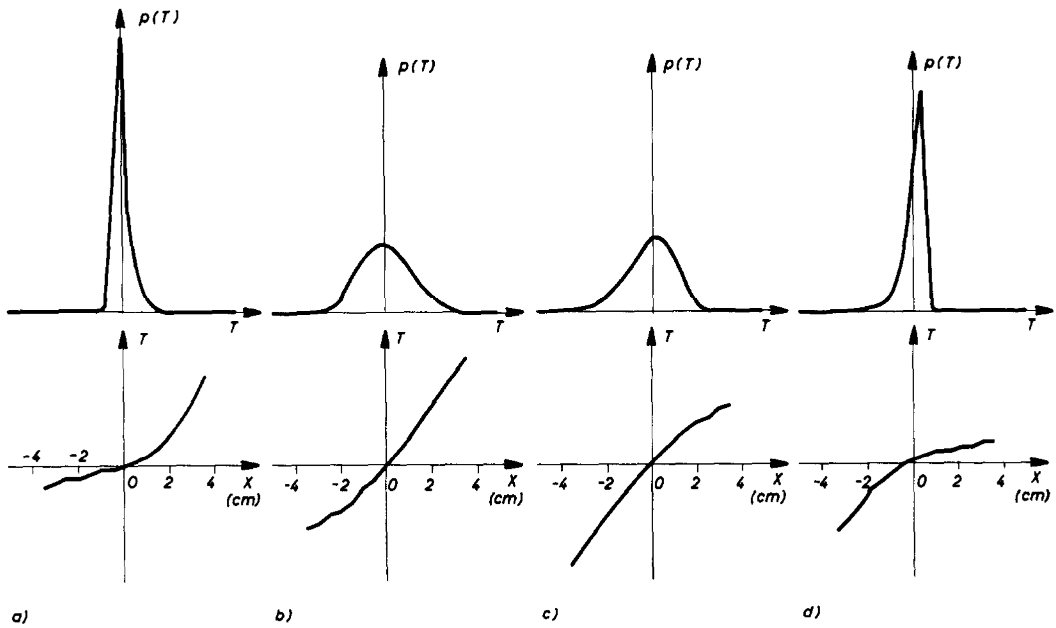

FIG. 5. Probability density functions $p(T)$ of measured temperature noise and the resulting profile curves $T(x)$.

be of about 1 cm. In the four diagrams $T(x)$ of Fig. 5 the temperature profile has always at $x = 0$ the value $T = 0$ (see Section I.3). With the measured mean temperature of one thermocouple it was possible to join all these four profiles together to the overall lateral profile plotted in Fig. 6. From this figure it follows that the real detection range of each of the four noise detectors was about 7 cm and hence much larger than the distance between two neighbouring detectors. In addition a good correspondence between the other four measured mean temperature values and the calculated profile can be observed. The value of 7 cm follows from the fact that $\delta = 1$ cm and that $T(\sigma_T)$ had been calculated in the range of $\pm 3.5\,\sigma_T$. This means that only 0.04% of the temperature eddies are coming from a region outside this lateral detection range of 7 cm.

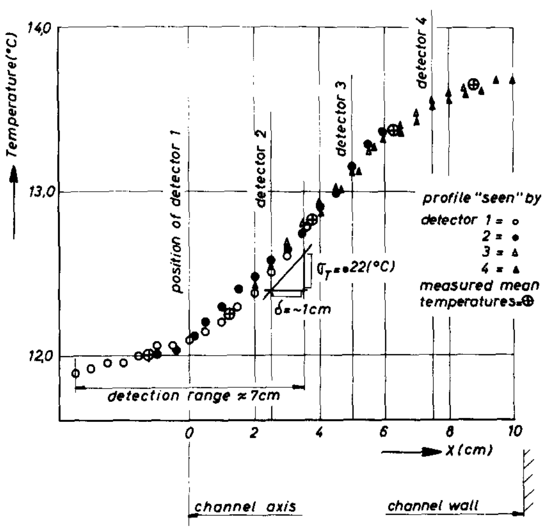

FIG. 6. Evaluated overall temperature profile in the test section.

Hence the detection range could only slightly be enlarged by increasing the confidence interval, i.e. for much longer measuring times.

Conclusion

The probability density function of temperature noise in a fluid flow is Gaussian in case of an underlying linear one-dimensional temperature profile. However, profiles with a laterally space-dependent gradient cause deviations from the normal distribution. Due to this fact it is possible to evaluate the one-dimensional lateral temperature profile from the amplitude distribution function of temperature measured at a fixed position and from the mean temperature at this position. This method can only work correctly if the profile is an unequivocal function of the lateral coordinate. The profile, however, can only be evaluated in a limited lateral range, which is given by the turbulent flow.

The proposed technique is an improvement of a method described in [10] and may also be applied in surveillance systems, e.g. for the malfunction detection in fuel elements of a nuclear power reactor.

This method can also be applied in case of homogenous pipe flow. In principle it should also be possible to evaluate properties of non-homogeneous turbulent fluid flow by inverse application of this technique.

Acknowledgement—The authors are indebted to Dr. W. Eifler and Dr. R. Nijsing for permission to use some of their unpublished experimental results.

REFERENCES

1. V. I. Subbotin, M. K. Ibragimov and E. V. Nomofilov, Statistical study of turbulent temperature pulsations in a liquid stream, *High Temperature* **2**, 59–64 (1964).
2. H. Fuchs. Wärmeübergang an strömen des Natrium. EIR-Bericht Nr. 241 (1973).

3. J. H. Rust and A. Sesonske, Turbulent temperature fluctuations in mercury and ethylene glycol in pipe flow, *Int. J. Heat Mass Transfer* **9**, 215–227 (1966).
4. G. K. Batchelor and A. A. Townsend, Turbulent diffusion, in *Surveys in Mechanics*, pp. 352–399, edited by G. K. Batchelor and R. W. Davies. Cambridge University Press, Cambridge (1956).
5. H. Nishihara and A. Ohtsubo, Measurement of temperature noise in an out-of-pile heat transfer loop, *J. Nucl. Energy* **23**, 99–111 (1969).
6. P. G. Bentley, The characteristics of temperature noise produced by turbulent mixing of a temperature profile, paper present at 4th Euromech. Colloquium on The Structure of Turbulence, Risley (1966).
7. R. Nijsing and W. Eifler, Private communication.
8. W. Eifler *et al.*, Report to be published.
9. E. Ohlmer, Sonden für die Messung von Temperaturschwankungen in Strömungen EUR-4291 (1975).
10. E. Ohlmer and D. Schwalm, The use of the amplitude distribution function of temperature noise for malfunction detection in a simulated reactor subassembly, *Atomkernenergie* **25**, 281–284 (1975).

DETERMINATION D'UN PROFIL DE TEMPERATURE UNIDIMENSIONNEL DANS UN ECOULEMENT TURBULENT A PARTIR DE LA DISTRIBUTION EN AMPLITUDE DES FLUCTUATIONS DE TEMPERATURE MESUREES EN UN POINT FIXE

Résumé—Dans un écoulement turbulent homogène les fluctuations de température sont engendrées par le mélange turbulent du fluide présentant un grandient latéral de température. Dans le cas d'un écoulement présentant un profil de température unidimensionnel dans la direction perpendiculaire à la vitesse du fluide, la distribution en amplitude des fluctuations de température est gaussienne si ce profil est linéaire, cela indépendamment de la valeur du gradient de température. Par contre, un profil à gradient variable produit des fluctuations non gaussiennes. De ce fait il est possible de déterminer le profil unidimensionnel à partir de la fonction densité de probabilité des fluctuations mesurée en un seul point de l'écoulement.

Une vérification expérimentale de cette méthode a été effectuée dans un écoulement d'eau pour $Re = 27\,000$.

La mesure d'un profil de température dans un fluide à l'aide d'une seule sonde fixe présente un intérêt pratique dans la détection d'anomalies de fonctionnement dans les assemblages combustibles des réacteurs nucléaires du type L M F B R.

DIE BESTIMMUNG EINES EINDIMENSIONALEN TEMPERATURPROFILS IN EINEM TURBULENTEN FLÜSSIGKEITSSTROM AUS DER AMPLITUDENVERTEILUNG DER AN EINER FIXIERTEN STELLE GEMESSENEN TEMPERATURSCHWANKUNGEN

Zusammenfassung—In einer homogenen turbulenten Flüssigkeitsströmung wurden Temperaturfluktuationen durch die Durchmischung der mit einem lateralen Temperaturprofil versehenen Flüssigkeit erzeugt. Ist dieses Temperaturprofil eindimensional und linear, dann besitzt das Temperaturrauschen eine Gauss-Verteilung—unabhängig vom numerischen Wert des Gradienten des Profils. Abweichungen von der Linearität des Profils erzeugen eindeutige Abweichungen von der Normalverteilung. Deshalb ist es möglich, das eindimensionale Profil aus der Amplitudenverteilungsdichte des gemessenen Temperaturrauschens zu ermitteln.

Versuche in einer turbulenten, homogenen Wasserströmung ($Re = 2700$) bestätigen die Brauchbarkeit der vorgeschlagenen Methode. Die Ermittlung eines Temperaturprofils mit einer festen Sonde ist von praktischem Interesse bei der Schadenfrüherkennung in Reaktorbrennelementen, wie sie bei schnellen Brütern benutzt werden.

ОПРЕДЕЛЕНИЕ ОДНОМЕРНОГО ТЕМПЕРАТУРНОГО ПРОФИЛЯ В ТЕМПЕРАТУРНОМ ПОТОКЕ ЖИДКОСТИ ПО АМПЛИТУДНОМУ РАСПРЕДЕЛЕНИЮ ИЗМЕРЕННЫХ ФЛУКТУАЦИЙ ПРИ ФИКСИРОВАННОМ ПОЛОЖЕНИИ ДЕТЕКТОРА

Аннотация — В турбулентном потоке однородной жидкости флуктуации температуры генерируются перемешиванием жидкости с поперечным температурным профилем. При течении жидкости с одномерным температурным профилем по нормали к скорости потока амплитудное распределение шумовой температуры в случае линейного профиля носит характер Гауссового распределения независимо от численной величины градиента. Отклонения от линейного профиля вызывают отклонения от нормального распределения. Благодаря этому, можно рассчитать основной одномерный профиль температуры с помощью функции плотности вероятности шума, измеренного в при фиксированном положении детектора в потоке жидкости. Опыты, проведенные с однородным турбулентным потоком воды ($Re = 27\,000$) подтвердили описанный метод.

Определение температурного профиля с помощью только одного детектора шума может представлять практический интерес при обнаружении неправильного срабатывания топливных элементов в ядерном реакторе типа реактор-размножителя на жидкометаллическом топливе.